as the minor premise, of course. But *B* is *A* is a *non sequitur*, both a formal and a material fallacy in the case. In fact, the instance is simply the common one for puzzling school boys.

It either rains or it does not rain.
It rains
∴ It does not rain.

The illusion is created by the failure to see that the principle of disjunction is not fulfilled by merely using the word 'not' before rains in the conclusion, when an additional negative is required by the dictum of this form of reasoning. The 'not' in this case is a part of the second term in the disjunction 'not rains,' and hence, when we follow the law of disjunctive inference, we should get 'It does not not rain,' or by double negatives 'It rains,' which is the true conclusion. So in Professor Jastrow's case. The *modus tollendo ponens* requires us to affirm the second term, which is 'not *A*,' and we get as the true conclusion *B* is not *A*, instead of *B* is *A*, which is a *nonsequitur*, as indicated.

But now, that I find that the conclusion is the same as the minor premise in the disjunctive reasoning, I may raise the further question whether there is not another material fallacy somewhere, since disjunctively I might get *B* is not *A*. In the instance before us this can be done, and in disjunctive inference the only fallacy that is most likely to occur is the *petitio principii*. The *non sequitur* will occur only when there is also a formal fallacy in it. Now, after assuming that *A* is *B*, it violates conversion to suppose that *B* is *A*, and it is a contradiction to suppose that *B* is not *A*. Hence with *A* is *B* as our premise, and *B* is either *A* or not *A* as the other; we have a *petitio principii* in the latter case. We might say that the disjunction is incomplete, which is possible if we assume that *A* is *B*, and which would only result in making the third alternative a particular proposition, *I* or *O*, with the formal fallacy mentioned in the prosyllogism, a *petitio principii* in the disjunctive syllogism, and a *non sequitur* in supposing that *B* is *A*.

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COLUMBIA UNIVERSITY,
NEW YORK, January 15, 1897.

SCIENTIFIC LITERATURE.

Higher Mathematics. A text-book for classical and engineering colleges. Edited by MANSFIELD MERRIMAN, Professor of Civil Engineering in Lehigh University, and ROBERT S. WOODWARD, Professor of Mechanics in Columbia University. New York, John Wiley & Sons. 1896. 8vo. Pp. xi+576.

The appearance of this rather unique volume is significant as a proof of the rapid development of mathematical instruction in this country. It is designed for undergraduates who have mastered the elements of the differential and integral calculus. After referring to the danger of excessive specialization and to the desirability of guiding the student to 'a comprehensive view of the mathematics of the present day,' the preface sets forth the general scope of this work in the following passage, which, for several reasons, is worth quoting in full: "During the past twenty years a marked change of opinion has occurred as to the aims and methods of mathematical instruction. The old ideas that mathematical studies should be pursued to discipline the mind, and that such studies were ended when an elementary course in the calculus had been covered, have for the most part disappeared. In our best classical and engineering colleges the elementary course in calculus is now given in the sophomore year, while lectures and seminary work in pure mathematics are continued during the junior and senior years. It is with the hope of meeting the existing demand for a suitable text to be used in such upper-class work that the editors enlisted the cooperation of the authors in the task of bringing together the chapters of the book." The following synopsis of the chapters will give some idea of the contents of 'Higher Mathematics:' I. 'The solution of equations,' by Mansfield Merriman (32 pp.); II. 'Determinants,' by Læenas Gifford Weld (37 pp.); III. 'Projective geometry,' by George Bruce Halsted (37 pp.); IV. 'Hyperbolic functions,' by James McMahon (62 pp.); V. 'Harmonic functions,' by William E. Byerly (57 pp.); VI. 'Functions of a complex variable,' by Thomas S. Fiske (77 pp.); VII. 'Differential equations,' by W. Woolsey Johnson (71 pp.); VIII. 'Grassmann's space analysis,' by Edward W.

Hyde (51 pp.); IX. 'Vector analysis and quaternions,' by Alexander Macfarlane (42 pp.); X. 'Probability and theory of errors,' by Robert S. Woodward (40 pp.); XI. 'History of modern mathematics,' by David Eugene Smith (63 pp.).

That this collection of comparatively brief and disconnected chapters, however well they may be written, could be used successfully as a text-book may appear doubtful. Most of the chapters are too short to serve as a satisfactory text for a college course. Nevertheless, the work is an exceedingly valuable one. The advantage to be gained by putting into the hands of the student a work covering so wide a range, in a form so attractive and easily accessible even without the assistance of a teacher, can hardly be overestimated. Both as an incentive to further study and as a book of reference, the volume will be of great service.

The proper selection and apportionment of subjects for such a general introduction to higher mathematics is a matter of great difficulty; on the whole, the selection has been made with excellent judgment. It is certainly to be regretted that the proposed chapter on elliptic functions had to be omitted; the subjects treated in chapters I., II., IV., VIII. and IX. would have been missed far less. Modern analytic geometry, the theory of substitutions and groups with its applications, non-Euclidean geometry, quantics and theoretical mechanics were probably excluded as too advanced or as not allowing of brief presentation.

From the point of view of pure mathematics the most interesting chapters in the book are Professor Fiske's 'Functions of a Complex Variable' and Professor Halsted's 'Projective Geometry.' The geometric mode of treatment which characterizes the first third of Professor Fiske's chapter will arouse the interest and self-activity of the student and thus prepare him for the more arduous analytical investigation of the critical points of the simplest monogenic functions which occupies the remainder of the chapter. The whole is written with the greatest care, and although this is the longest chapter in the book one cannot help regretting that it is not longer. In but one or two cases conciseness seems to be carried so far as to endanger clearness—for instance, in the definition of uniform convergence (p. 274); but in general the presentation is as clear as it is precise.

Professor Halsted gives us a carefully worked-out exposition of von Staudt's system of synthetic geometry. The logical development, as was to be expected, is admirable; the form of presentation is exceedingly concise and neat. A mathematician familiar with the subject and with von Staudt's terminology may read this chapter with pleasure. But the beginner, for whom this volume is intended, will be sorely perplexed. Even if he has energy and patience enough to learn the new language here spoken, and comes to understand such phrases as "A tetrastim with dots in a conic has each pair of codots costraight with a pair of fanpoints of the tetragram of tangents at the dots" (Art. 91, p. 85), or "Two correlated polystims whose paired dots and codots have their joins copunctal are called 'coplanar'" (Art. 51, p. 76), of what use is this to him? Few persons will understand him, and he himself will be unable to understand the masters who have written, and are still writing, on the science of projective geometry. But, even apart from this passion for coining new words, it seems to the writer that the rigid formality and exclusiveness of the treatment here adopted tends to make a naturally easy and attractive subject unnecessarily difficult and almost forbidding to the beginner, and to give him a one-sided idea of what is now meant by projective geometry. To mention a minor point, a reference to von Staudt's 'Geometrie der Lage,' which, by a curious oversight, is nowhere mentioned, would have been in place in connection with the 'fundamental theorem' of Art. 59 (p. 77), which corresponds to von Staudt's Art. 88. The printer is probably responsible for assigning Pappus to the age of Plato (p. 104).

To the student of applied mathematics the chapters on 'Harmonic Functions' and on 'Probability and Theory of Errors' will prove of most value. The first half of Professor Woodward's chapter treats of the theory of probability proper, beginning with permutations and leading up to Bernoulli's theorem; the latter half, on 'laws of error,' is particularly valuable as embodying the results

of the author's own investigations on the errors of interpolated values. This chapter will form an excellent supplement to a course on the method of least squares.

Professor Byerly's chapter on harmonic functions is a model of clear and attractive exposition, in a subject by no means easy of approach to the beginner. It is, of course, largely based on the author's more extensive text-book. After showing on three particular examples how the attempt to solve certain physical problems naturally leads to Fourier series, to zonal and cylindrical harmonics, the author discusses each of these three subjects in some detail, illustrating every method by numerical examples, some of which are worked out even to the arrangement of the logarithmic work. Nothing could be more useful to the student of applied mathematics, while the pure mathematician may regret that the constant occupation with methods of solving certain problems leaves no room for inquiring into the real nature and characteristics of the functions under discussion. But, in a brief chapter, more than is here given could hardly be expected.

The introduction of numerous applications and exercises, which is a general feature of this volume, is also very prominent in Professor McMahon's chapter on hyperbolic functions. This chapter is perhaps more complete in itself than any other chapter in the book. It gives a very satisfactory exposition of the theory, with graphical representations, seven pages of tables and well-chosen applied problems.

Professor Johnson's excellent chapter on differential equations is naturally one of the longest in the book and also attains a certain degree of completeness.

On the other hand, Professor Merriman's chapter on 'the solution of equations' appears rather meagre, perhaps, because the author, as one of the editors, felt bound to keep strictly within the prescribed limits of space. A somewhat remarkable statement about the impossibility of the algebraic solution of the general quintic appears at the bottom of p. 22. After referring briefly to the researches of Abel and Galois, the author says: "Although these discussions are complex and not devoid of doubt," (a foot-note gives an inaccurate reference to Kronecker and a reference to Cockle), "they have been generally accepted as conclusive. Moreover, the fact that the quintic is still unsolved, in spite of the enormous amount of work done upon it during the past two centuries, is strong evidence that the problem is an impossible one." Comment is unnecessary.

The chapter on determinants contains more than might be expected from its brevity. Professor Weld's modesty in not referring anywhere to his text-book on the subject is worthy of mention.

The geometrical calculus is represented by two interesting chapters. The elements of Grassmann's methods as applied to plane and solid geometry are set forth at some length by Professor Hyde, while Professor Macfarlane treats of vector addition and multiplication, with particular reference to their application in mathematical physics. The quaternion proper, although it figures in the title of Chapter IX., receives but slight attention. Both chapters are far too brief to show the real power of these methods, which appears especially when geometrical differentiation and integration are introduced. The present writer cannot help regretting that Professor Hyde has not adopted the remarkably elegant and simple treatment of Grassmann's fundamental ideas proposed by Peano. From the point of view of pure mathematics Peano's method of laying the foundation for a geometrical calculus can hardly be improved upon. The physicist, however, will probably, for some time to come, prefer to become acquainted with vector analysis in close connection with the development of his physical and mechanical notions, in a manner similar to that pursued by Oliver Heaviside in his 'Electro-magnetic Theory,' Vol. I. (1893). Fortunately, Professor Macfarlane's methods and notations do not seem to differ now very much from Mr. Heaviside's. The peculiarly cumbrous notation for what might be called the polar coordinates of a vector is an exception.

In the last chapter Professor D. E. Smith gives a rapid survey of the historical development of the various branches of mathematics during the nineteenth century. This rather difficult task seems to be accomplished in a very satisfactory way, the chapter being evidently

based upon the best sources and made with great care. The chapter adds much to the value of this volume as a book of reference.

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The Development of the Periodic Law. By F. P. VENABLE, Ph. D., Professor in the University of North Carolina. Easton, Pa., Chemical Publishing Company. 1896. Pp. viii+321. Price, $2.50.

The purpose of this book cannot be better given than in the author's own words: "This work * * * is to be used for purposes of reference and of study, and not as a mere history of the subject. The errors and repetitions of the writers upon this subject in the past few years have abundantly proved the necessity for some such gathering and systematizing the work of former years."

Professor Venable's work in writing his recently published History of Chemistry has given him an excellent preparation for the critical study of the discovery and development of the periodic law, which is given in this volume. As stated by the author, much of the literature of the subject is in hidden and out-of-the-way places and a very real service is rendered to chemical science in thus coordinating it and making it more easily accessible. The scope of the book includes an account of the numerous attempts which have been made to discover numerical and other relations between the atomic weights and also an account of speculations as to the origin of the elements and their relation to some fundamental form of matter.

Calculations and speculations of this kind have had a fatal fascination for a great many chemists, and as we look over the literature and see how much has been written that is fanciful, and how much that in the light of better knowledge has been found erroneous and worthless, we are almost tempted to turn from the whole subject in disgust. And there is no doubt that many of these speculations have been worthless and the time of their authors has been nearly or quite wasted, for they have led to no accepted conclusions and they have given no incentive to useful work. But the periodic system stands on quite a different plane, for it furnishes us the best means at present available for coordinating our knowledge of the chemical elements, and it has furnished the incentive for a large amount of most excellent experimental work. That there are some imperfections in the system and that it does not, at present, give any accurate mathematical expression for our chemical knowledge must be admitted. It is tantalizing in its suggestiveness, and most chemists believe that it half reveals facts which will be of profound importance when fully understood. If the present work turns the attention of chemists in that direction it may prove very useful.

A quite full bibliography and an excellent index add to the usefulness of the work.

W. A. N.

Notes on Qualitative Analysis, arranged for the use of students of the Rensselaer Polytechnic Institute. By W. P. MASON, Professor of Chemistry. Third Edition. Easton, Pa., Chemical Publishing Company. 1896. Pp. 56. Price, 80 cents.

This book gives a concise statement of the more important qualitative tests for metals and acids, those for the metals being arranged in the order of Fresenius. Then follow tables for analysis of metals, and five pages giving very short directions for the analysis of alloys, insoluble substances and alkaline solutions.

The selection of tests is satisfactory and the book will, doubtless, furnish a basis for a good short course in the subject. It would seem, however, that even an elementary work should give directions which are reliable for cases of very common occurrence. For instance, ammonia often fails to separate small quantities of silver chloride from mercurous chloride; and ammonia will not separate zinc from chromium unless the zinc is in excess. Neither case is provided for in the directions given.

Books of this character may furnish students with excellent drill in scientific methods of work and, in the hands of a good teacher, are satisfactory from that standpoint, but the student should understand that he is liable to fall into very serious mistakes if he attempts to use the directions for practical work.

The references to Watts' dictionary and the